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THE ALLOCATION OF RESOURCES IN STEADY-STATE UNBALANCED GROWTH

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THE ALLOCATION OF RESOURCES IN
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By HANS BREMS*

With few exceptions, modern growth models are models of steady-state and balanced¹ growth of homogeneous consumption and capital stock, hence miss imbalance [1], [6] as well as the allocation of resources.

To allow for imbalance, a growth model needs at least two goods. But to allow for the allocation of resources, the two goods cannot be the consumers' good and the producers' good found in the usual [5] two-sector growth models. With only one consumers' good, such models are still models of homogeneous consumption, permitting no substitution among consumers' goods and asking no question, hence offering no answer, concerning the allocation of consumption

expenditure among consumers goods. With only one producers' good such models are still models of homogeneous capital stock, permitting no substitution among producers' goods and asking no question, hence offering no answer, concerning the allocation of investment expenditure among producers' goods.

We wish to build the simplest possible growth model of heterogeneous consumption as well as capital stock, thus allowing for the full allocation of resources. To do that we assume each of our two goods to serve interchangeably as a consumers' or as a producers' good: The physical output of the j th good is X_j where $j = 1, 2$. The j th good is produced from labor L_j and two immortal capital stocks S_{ij} where $i = 1, 2$. There are, then, four capital stocks S_{ij} and four investments I_{ij} in our model. Between two such industries we specify a fourfold interaction:

The two industries compete in their demand for labor. In the

labor market they must pay the same money wage rate w , a parameter. Goods prices P_j are variables, hence the real wage rate w/P_j is also a variable.

The two industries compete in their demand for investment goods. In the market for the j th good they must pay the same price P_j . A firm producing the j th good and setting aside part of its own output for investment I_{jj} should charge itself the price P_j as an opportunity cost.

The two industries compete in their demand for money capital. In the money-capital market the capitalist-entrepreneurs allocate their savings between the two industries such as to maximize the present worth of all their future profits.

The two industries compete in their supply of consumers' goods. In the consumers' goods market the two goods are good, but not perfect, substitutes, and each consumer has a taste for both of them.

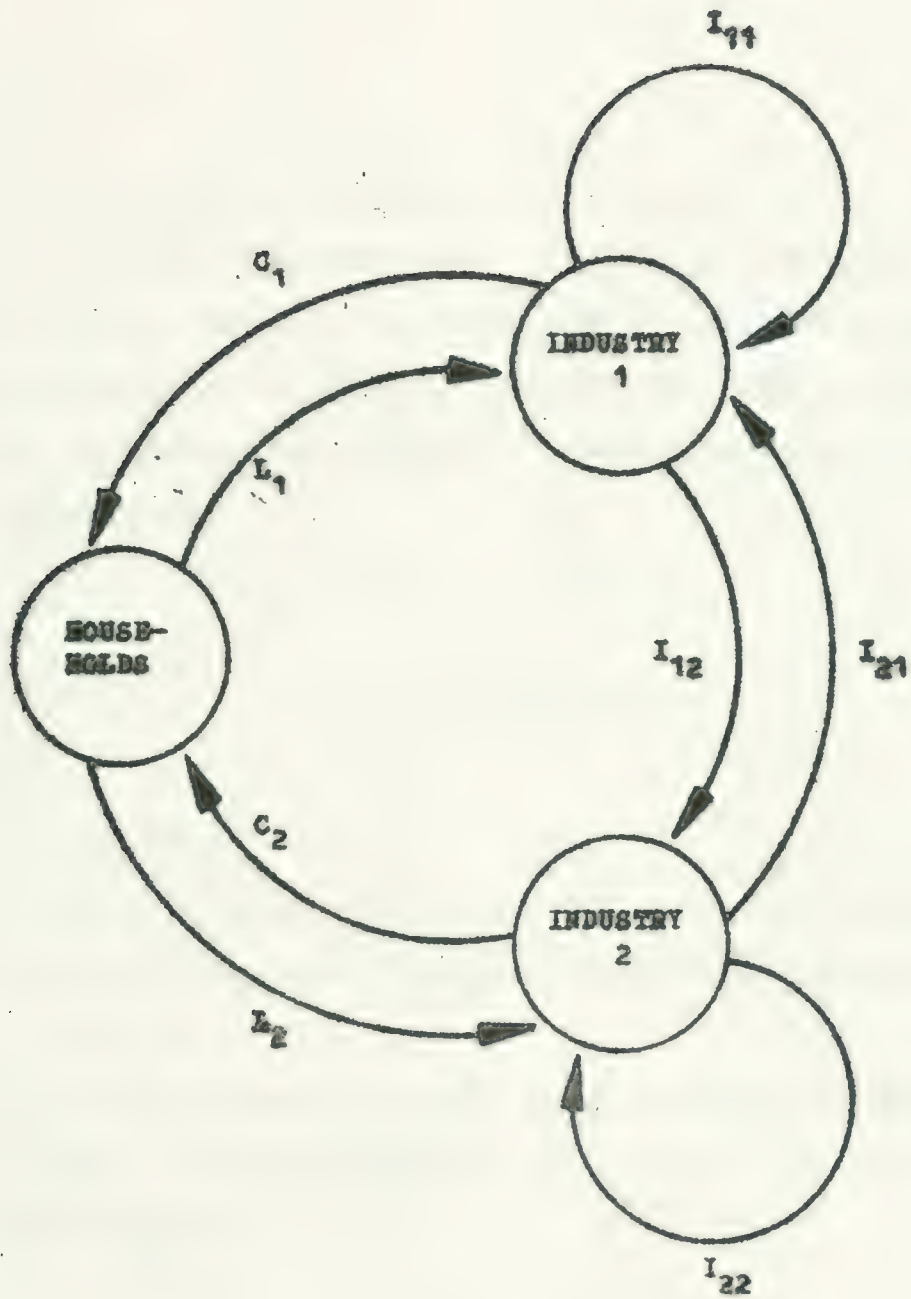


FIGURE 1. THE EIGHT PHYSICAL FLOWS

Figure 1 shows all physical flows in our model. Section I defines variables and parameters. Section II specifies the model mathematically. Section III finds the equilibrium solutions for proportionate rates of growth. Section IV finds the equilibrium solutions for levels of variables. Certain proofs are banished to two appendices.

I. NOTATION

Variables

C \equiv consumption

ϕ \equiv function to be maximized by the Lagrange-multiplier method

g_v \equiv proportionate rate of growth of variable v where $v \equiv C, I, L, P, S, X,$ and Y

I_{ij} \equiv investment of output of i th industry in j th industry

κ_{ij} \equiv physical marginal productivity of capital stock S_{ij}

L \equiv labor employed

P \equiv price of good
 S_{ij} \equiv jth industry's physical capital stock of ith industry's good
 U \equiv utility
 W \equiv wage bill
 X \equiv physical output
 Y \equiv national money income
 Z \equiv profits bill
 ζ \equiv present worth of all future profits bills

Parameters

A \equiv exponent of individual utility function
 α, β \equiv exponents of production function
 c \equiv propensity to consume national money income
 e \equiv Euler's number, the base of natural logarithms
 F \equiv available labor force
 g_p \equiv proportionate rate of growth of parameter p where $p \equiv F, M,$

and w

- λ \equiv Lagrange multiplier
- M \equiv multiplicative factor of production function
- N \equiv multiplicative factor of individual utility function
- r \equiv discount rate applied by capitalist-entrepreneurs
- w \equiv money wage rate

The parameters listed are stationary except F , M , and w , whose growth rates g_F , g_M , and g_w are stationary.

The symbol π_i to be defined in Section II; h_j in Section IV, 2; ρ , m , n , and μ_i in Section IV, 3; v_{ij} and ξ_i in Section IV, 5; and ψ in Appendix I all stand for agglomerations of parameters and variables. Symbols t and τ are time coordinates. Subscripts $i = 1, 2$ and $j = 1, 2$ refer to industry number. All flow variables refer to the instantaneous rate of that variable measured on a per annum basis.

• *Journal of the American Medical Association*, 1997; 277: 1001-1005

II. THE EQUATIONS OF THE MODEL

17 variable growth rates are listed in Section I, i. e. two growth rates of each of C_i , L_i , P_i , and X_i ; four growth rates of each of I_{ij} and S_{ij} ; and one growth rate of Y . To all apply the definition

$$(1) \text{ through } (17) \quad g_v \equiv \frac{dv}{dt} \frac{1}{v}$$

Define investment as the derivative of capital stock with respect to time

$$(18) \text{ through } (21) \quad I_{ij} \equiv \frac{dS_{ij}}{dt}$$

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...and the fact that the *in vitro* and *in vivo* results are in good agreement.

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Let the j th industry apply the Cobb-Douglas production function

$$(22) \quad X_1 = M_1 L_1^{\alpha_1} S_{11}^{\beta_{11}} S_{21}^{\beta_{21}}$$

$$(23) \quad X_2 = M_2 L_2^{\alpha_2} S_{12}^{\beta_{12}} S_{22}^{\beta_{22}}$$

where $0 < \alpha_j < 1$; $0 < \beta_{ij} < 1$; $\alpha_1 + \beta_{11} + \beta_{21} = 1$; $\alpha_2 + \beta_{12} + \beta_{22} = 1$; and $M_j > 0$. In each industry let profit maximization under pure competition equalize real wage rate and physical marginal productivity of labor:

$$(24), (25) \quad \frac{w}{P_j} = \frac{\partial X_j}{\partial L_j} = \alpha_j \frac{X_j}{L_j}$$

Physical marginal productivities of capital at time t are

$$(26) \text{ through } (29) \quad \kappa_{ij}(t) \equiv \frac{\partial X_j(t)}{\partial S_{ij}(t)} = \beta_{ij} \frac{X_j(t)}{S_{ij}(t)}$$

Multiply (26) through (29) by price of output of j th industry $P_j(t)$ to find value marginal productivities of capital at time t . Define money profits earned at time t on each physical unit of capital stock $S_{ij}(t)$ as its value marginal productivity. Then multiply by $S_{ij}(t)$ to find money profits earned at time t on capital stock $S_{ij}(t)$. Sum over $i = 1, 2$ and define the outcome as money profits earned at time t on whatever capital stock exists at that time in the entire j th industry:

$$(30), (31) \quad Z_j(t) \equiv \sum_{i=1}^2 \kappa_{ij}(t) P_j(t) S_{ij}(t) = P_j(t) X_j(t) \sum_{i=1}^2 \beta_{ij}$$

- 11 -

Sum over $j = 1, 2$ and define the outcome as money profits earned at time t on whatever capital stock exists at that time in the entire economy:

$$(32) \quad Z(t) \equiv \sum_{j=1}^2 Z_j(t)$$

As seen from the present time τ this profits bill is $Z(t)e^{-r(t - \tau)}$ where e is Euler's number, the base of natural logarithms, and r is the discount rate applied by the capitalist-entrepreneurs. Finally integrate this over $t = \tau$ through ∞ and define the outcome as the present worth of all future profits bills

$$(33) \quad \zeta(\tau) \equiv \int_{\tau}^{\infty} Z(t)e^{-r(t - \tau)} dt$$

Now let capitalist-entrepreneurs use their control variable I_{ij} to optimize the allocation of their capital stock

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$$\log_{10} \frac{1}{100} = -2 \quad (100)$$

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$$\log_{10} \frac{1}{100} = -2 \quad (100)$$

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S_{ij} within as well as between industries. Within the j th industry they act as stockholders optimizing S_{ij} where $i = 1, 2$ by appointing the right managers. Between industries they act as stockholders optimizing S_{ij} where $j = 1, 2$ by purchasing stock in the right industry. "Optimizing" in what sense? In the sense that

$$(34) \quad \zeta(\tau) = \text{maximum}$$

Under full employment, available labor force must equal the sum of labor employed by the two industries:

$$(35) \quad F = \sum_{i=1}^2 L_i$$

Define the wage bill as the money wage rate times employment:

Page 1

It is also well known that the industry
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there is a great difference. In the case
of the industry, the right to work in is
very different from the one that the right to work in.

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$$(36) \quad W \equiv w \sum_{i=1}^2 L_i$$

Define national money income as the sum of the wage bill and the profits bill:

$$(37) \quad Y \equiv W + Z$$

Let all persons have the same utility function. Let the utility function of the k th person be

$$U_k = N C_{1k}^{A_1} C_{2k}^{A_2}$$

where $0 < A_i < 1$ and $N > 0$. Let there be s persons, and let the k th person's money income be Y_k where

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$$\sum_{k=1}^S Y_k = Y$$

Let all persons spend the fraction c , where $0 < c < 1$, of their money income. Then the budget constraint of the k th person is

$$cY_k = P_1 C_{1k} + P_2 C_{2k}$$

Maximize the k th person's utility subject to his budget constraint and find his two demand functions. Then add the s individual demand functions for each good and find the two Graham [2] aggregate demand functions

$$(38), (39) \quad C_i = \pi_i Y / P_i$$

where

- 15 -

$$\pi_i = \frac{cA_i}{A_1 + A_2}$$

Industry output equilibrium requires the output of the i th industry to equal the sum of consumption and investment demand for it, or inventory would either accumulate or be depleted:

$$(40), (41) \quad x_i = c_i + \sum_{j=1}^2 I_{ij}$$

III. SOLUTIONS FOR PROPORTIONATE RATES OF GROWTH

Our system (1) through (41) possesses the following set of steady-state solutions for its equilibrium proportionate rates of growth:

$$(42), (43) \quad g_{Ci} = g_{Xi}$$

$$(44) \text{ through } (47) \quad g_{Iij} = g_{Xi}$$

$$(48), (49) \quad g_{Li} = g_F$$

$$(50) \quad g_{P1} = g_w - \frac{(1 - \beta_{22})g_{M1} + \beta_{21}g_{M2}}{(1 - \beta_{11})(1 - \beta_{22}) - \beta_{12}\beta_{21}}$$

$$(51) \quad g_{P2} = g_w - \frac{(1 - \beta_{11})g_{M2} + \beta_{12}g_{M1}}{(1 - \beta_{11})(1 - \beta_{22}) - \beta_{12}\beta_{21}}$$

$$(52) \text{ through } (55) \quad g_{Sij} = g_{Xi}$$

[illegible]

$$(56) \quad g_{X1} = \frac{(1 - \beta_{22})g_{M1} + \beta_{21}g_{M2}}{(1 - \beta_{11})(1 - \beta_{22}) - \beta_{12}\beta_{21}} + g_F$$

$$(57) \quad g_{X2} = \frac{(1 - \beta_{11})g_{M2} + \beta_{12}g_{M1}}{(1 - \beta_{11})(1 - \beta_{22}) - \beta_{12}\beta_{21}} + g_F$$

$$(58) \quad g_Y = g_F + g_W$$

To see that it does, the reader should take derivatives with respect to time of all equations (18) through (41) except (26) through (29) and (33), (34). He should then use definitions (1) through (17), insert solutions (42) through (58), and convince himself that each equation is satisfied.

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21. The twenty-first part of the report
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 25. The twenty-fifth part of the report
 26. The twenty-sixth part of the report
 27. The twenty-seventh part of the report
 28. The twenty-eighth part of the report
 29. The twenty-ninth part of the report
 30. The thirtieth part of the report

We defined balanced growth as identical proportionate rates of growth of physical output for all goods. According to our solutions (42) through (58), is our steady-state growth balanced or unbalanced?

Growth does spill over from one industry to the other. For example, according to (44) through (47) a more rapidly growing industry i would transmit some of its growth to a more slowly growing industry j investing in the i th industry's good. But the spillover is normally not enough to generate balanced growth. Use (56), (57), and the assumptions that $\alpha_1 + \beta_{11} + \beta_{21} = 1$ and $\alpha_2 + \beta_{12} + \beta_{22} = 1$ to find that

$$g_{X1} \geq g_{X2} \text{ implies } g_{M1}/g_{M2} \geq \alpha_1/\alpha_2,$$

respectively. Or in English: The first industry's physical output may grow more rapidly than that of the second industry for two and only² two reasons, i. e., first if everything else being equal the

Chapter 1

The first step in the process of understanding a system is to identify the components and their interactions. This involves a thorough examination of the system's structure and function. The next step is to define the system's boundaries and the scope of the study. This is followed by a detailed analysis of the system's behavior and the identification of the key variables and parameters. The final step is to develop a model of the system that can be used to predict its behavior under various conditions. This model is then validated against experimental data to ensure its accuracy and reliability.

$$V_{\text{eff}} = \frac{1}{2} m \omega^2 r^2 + \frac{1}{2} m \omega^2 z^2$$

The effective potential energy V_{eff} is a function of the radial coordinate r and the vertical coordinate z . It represents the total energy of the system, taking into account the kinetic energy of the particles and the potential energy of the external fields. The effective potential energy is used to describe the motion of the particles in the system and to determine the equilibrium positions and the stability of the system.

first industry has more rapid technological progress g_{M_i} than the second industry, second, if everything else being equal the physical output of the first industry has a lower labor elasticity α_i than that of the second industry: The less labor-sensitive industry is less hampered by the fact that under technological progress labor force is growing less rapidly than physical capital stocks.

It does, however, follow from (50), (51), (56), and (57) that unlike physical outputs X_i , industry revenues $P_i X_i$ will grow at the same proportionate rate $g_F + g_W$.

IV. SOLUTIONS FOR LEVELS

So much for proportionate rates of growth. Let us now turn to the allocation of resources and solve for the allocation of savings between industries; the levels of industry revenues; employments; national money income; physical outputs; prices; physical

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capital stocks and their physical marginal productivities;
consumption; and income distribution.

1. Saving Equals Investment

Use (24), (25), and (36) to see that $W = \alpha_1 P_1 X_1 + \alpha_2 P_2 X_2$, and
(30) through (32) to see that $Z = (\beta_{11} + \beta_{21})P_1 X_1 + (\beta_{12} + \beta_{22})P_2 X_2$,
hence national income equals national output:

$$(59) \quad Y = P_1 X_1 + P_2 X_2$$

Multiply (40) and (41) by P_1 and P_2 , respectively, insert (38),
(39), and (59) and find saving to equal investment:

$$(60) \quad (1 - c)Y = P_1(I_{11} + I_{12}) + P_2(I_{21} + I_{22})$$

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$$\frac{1}{X_1} + \frac{1}{X_2} = 1$$

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$$\frac{1}{X_1} + \frac{1}{X_2} = 1$$

2. Present-Worth Maximization

Subject to the constraint (60) let the capitalist-entrepreneurs use their control variable I_{ij} to optimize the allocation of their capital stock S_{ij} within as well as between industries. "Optimize" in what sense? In the sense of maximizing the present worth $\zeta(\tau)$ of all future profits bills in accordance with (34). Using (30) through (33) we write present worth as

$$\zeta(\tau) = \int_{\tau}^{\infty} [(\beta_{11} + \beta_{21})P_1(t)X_1(t) + (\beta_{12} + \beta_{22})P_2(t)X_2(t)]e^{-r(t - \tau)}dt$$

Let it be foreseen by the capitalist-entrepreneurs that prices are growing in accordance with our steady-state solutions (50) and (51), hence

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$$P_j(t) = e^{g_{Pj}(t - \tau)} P_j(\tau)$$

and that outputs are growing in accordance with our steady-state solutions (56) and (57), hence

$$X_j(t) = e^{g_{Xj}(t - \tau)} X_j(\tau)$$

Consequently we may take prices and outputs outside the integral sign and write present worth as

$$\begin{aligned} \zeta(\tau) = & (\beta_{11} + \beta_{21})P_1(\tau)X_1(\tau) \int_{\tau}^{\infty} e^{(g_{P1} + g_{X1} - r)(t - \tau)} dt + \\ & (\beta_{12} + \beta_{22})P_2(\tau)X_2(\tau) \int_{\tau}^{\infty} e^{(g_{P2} + g_{X2} - r)(t - \tau)} dt \end{aligned}$$

Since in this expression all variables refer to the same time

τ , we may purge it of τ . Use (50), (51), (56), and (57) to see that $g_{pj} + g_{Xj} = g_F + g_W$. Assume that $g_F + g_W < r$, then integrate:

$$\zeta = \frac{(\beta_{11} + \beta_{21})P_1X_1 + (\beta_{12} + \beta_{22})P_2X_2}{r - (g_F + g_W)}$$

Inserting (30) through (32) into this we find the simple relationship between profits and present worth under steady-state growth:

$$(61) \quad Z = [r - (g_F + g_W)]\zeta$$

Maximizing present worth ζ subject to the constraint (60) is most easily done by using a Lagrange multiplier: Define a new function to be maximized

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Let $f(x) = (x^2 + 1)^2$. Then $f'(x) = 2(x^2 + 1) \cdot 2x = 4x(x^2 + 1)$.
 The derivative of $f(x)$ is $f'(x) = 4x(x^2 + 1)$.

$$\frac{d}{dx} \left(\frac{x^2 + 1}{x^2 - 1} \right) = \frac{(x^2 - 1) \cdot 2x - (x^2 + 1) \cdot 2x}{(x^2 - 1)^2} = \frac{2x(x^2 - 1 - x^2 - 1)}{(x^2 - 1)^2} = \frac{-4x}{(x^2 - 1)^2}$$

Let $f(x) = \sin(x)$. Then $f'(x) = \cos(x)$.
 The derivative of $f(x)$ is $f'(x) = \cos(x)$.

$$\frac{d}{dx} \left(\frac{1}{x} \right) = -\frac{1}{x^2}$$

Let $f(x) = \ln(x)$. Then $f'(x) = \frac{1}{x}$.
 The derivative of $f(x)$ is $f'(x) = \frac{1}{x}$.

$$\phi \equiv \zeta + \lambda[(1 - c)Y - P_1(I_{11} + I_{12}) - P_2(I_{21} + I_{22})]$$

What to do with Y? Insert (61) into (37), insert the outcome into ϕ and write the latter

$$(62) \quad \phi = \{1 + \lambda(1 - c)[r - (g_F + g_W)]\}\zeta +$$

$$\lambda(1 - c)W - \lambda[P_1(I_{11} + I_{12}) + P_2(I_{21} + I_{22})]$$

The first four first-order conditions for a maximum ϕ are

$$(63) \quad \frac{\partial \phi}{\partial I_{ij}} = h_j \frac{\partial X_j}{\partial I_{ij}} - \lambda P_i = 0$$

where

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$$h_j \equiv \frac{\{1 + \lambda(1 - c)[r - (g_F + g_w)]\}(\beta_{1j} + \beta_{2j})P_j}{r - (g_F + g_w)}$$

Now according to the production functions (22) and (23), output X_j is a function of capital stock S_{ij} rather than of investment I_{ij} . But according to (1) through (21)

$$(64) \quad S_{ij} \equiv I_{ij}/g_{Sij}$$

where our steady-state growth, as specified by (52) through (57), permits us to express g_{Sij} solely in terms of parameters. Inserting (64) into the production functions (22) and (23) we find

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Figure 1. The effect of the concentration of the *Agrobacterium* suspension on the transformation efficiency of *Agrobacterium* strains. The concentration of the *Agrobacterium* suspension was 10⁶ cells/ml (A), 10⁷ cells/ml (B), 10⁸ cells/ml (C), and 10⁹ cells/ml (D). The concentration of the *Agrobacterium* suspension was 10⁶ cells/ml (A), 10⁷ cells/ml (B), 10⁸ cells/ml (C), and 10⁹ cells/ml (D). The concentration of the *Agrobacterium* suspension was 10⁶ cells/ml (A), 10⁷ cells/ml (B), 10⁸ cells/ml (C), and 10⁹ cells/ml (D). The concentration of the *Agrobacterium* suspension was 10⁶ cells/ml (A), 10⁷ cells/ml (B), 10⁸ cells/ml (C), and 10⁹ cells/ml (D).

1996, 1997, 1998, 1999, 2000, 2001, 2002, 2003, 2004, 2005, 2006, 2007, 2008, 2009, 2010, 2011, 2012, 2013, 2014, 2015, 2016, 2017, 2018, 2019, 2020, 2021, 2022, 2023, 2024, 2025, 2026, 2027, 2028, 2029, 2030, 2031, 2032, 2033, 2034, 2035, 2036, 2037, 2038, 2039, 2040, 2041, 2042, 2043, 2044, 2045, 2046, 2047, 2048, 2049, 2050, 2051, 2052, 2053, 2054, 2055, 2056, 2057, 2058, 2059, 2060, 2061, 2062, 2063, 2064, 2065, 2066, 2067, 2068, 2069, 2070, 2071, 2072, 2073, 2074, 2075, 2076, 2077, 2078, 2079, 2080, 2081, 2082, 2083, 2084, 2085, 2086, 2087, 2088, 2089, 2090, 2091, 2092, 2093, 2094, 2095, 2096, 2097, 2098, 2099, 2100, 2101, 2102, 2103, 2104, 2105, 2106, 2107, 2108, 2109, 2110, 2111, 2112, 2113, 2114, 2115, 2116, 2117, 2118, 2119, 2120, 2121, 2122, 2123, 2124, 2125, 2126, 2127, 2128, 2129, 2130, 2131, 2132, 2133, 2134, 2135, 2136, 2137, 2138, 2139, 2140, 2141, 2142, 2143, 2144, 2145, 2146, 2147, 2148, 2149, 2150, 2151, 2152, 2153, 2154, 2155, 2156, 2157, 2158, 2159, 2160, 2161, 2162, 2163, 2164, 2165, 2166, 2167, 2168, 2169, 2170, 2171, 2172, 2173, 2174, 2175, 2176, 2177, 2178, 2179, 2180, 2181, 2182, 2183, 2184, 2185, 2186, 2187, 2188, 2189, 2190, 2191, 2192, 2193, 2194, 2195, 2196, 2197, 2198, 2199, 2200, 2201, 2202, 2203, 2204, 2205, 2206, 2207, 2208, 2209, 2210, 2211, 2212, 2213, 2214, 2215, 2216, 2217, 2218, 2219, 2220, 2221, 2222, 2223, 2224, 2225, 2226, 2227, 2228, 2229, 2230, 2231, 2232, 2233, 2234, 2235, 2236, 2237, 2238, 2239, 2240, 2241, 2242, 2243, 2244, 2245, 2246, 2247, 2248, 2249, 2250, 2251, 2252, 2253, 2254, 2255, 2256, 2257, 2258, 2259, 2260, 2261, 2262, 2263, 2264, 2265, 2266, 2267, 2268, 2269, 2270, 2271, 2272, 2273, 2274, 2275, 2276, 2277, 2278, 2279, 2280, 2281, 2282, 2283, 2284, 2285, 2286, 2287, 2288, 2289, 2290, 2291, 2292, 2293, 2294, 2295, 2296, 2297, 2298, 2299, 2300, 2301, 2302, 2303, 2304, 2305, 2306, 2307, 2308, 2309, 2310, 2311, 2312, 2313, 2314, 2315, 2316, 2317, 2318, 2319, 2320, 2321, 2322, 2323, 2324, 2325, 2326, 2327, 2328, 2329, 2330, 2331, 2332, 2333, 2334, 2335, 2336, 2337, 2338, 2339, 2340, 2341, 2342, 2343, 2344, 2345, 2346, 2347, 2348, 2349, 2350, 2351, 2352, 2353, 2354, 2355, 2356, 2357, 2358, 2359, 2360, 2361, 2362, 2363, 2364, 2365, 2366, 2367, 2368, 2369, 2370, 2371, 2372, 2373, 2374, 2375, 2376, 2377, 2378, 2379, 2380, 2381, 2382, 2383, 2384, 2385, 2386, 2387, 2388, 2389, 2390, 2391, 2392, 2393, 2394, 2395, 2396, 2397, 2398, 2399, 2400, 2401, 2402, 2403, 2404, 2405, 2406, 2407, 2408, 2409, 2410, 2411, 2412, 2413, 2414, 2415, 2416, 2417, 2418, 2419, 2420, 2421, 2422, 2423, 2424, 2425, 2426, 2427, 2428, 2429, 2430, 2431, 2432, 2433, 2434, 2435, 2436, 2437, 2438, 2439, 2440, 2441, 2442, 2443, 2444, 2445, 2446, 2447, 2448, 2449, 2450, 2451, 2452, 2453, 2454, 2455, 2456, 2457, 2458, 2459, 2460, 2461, 2462, 2463, 2464, 2465, 2466, 2467, 2468, 2469, 2470, 2471, 2472, 2473, 2474, 2475, 2476, 2477, 2478, 2479, 2480, 2481, 2482, 2483, 2484, 2485, 2486, 2487, 2488, 2489, 2490, 2491, 2492, 2493, 2494, 2495, 2496, 2497, 2498, 2499, 2500, 2501, 2502, 2503, 2504, 2505, 2506, 2507, 2508, 2509, 2510, 2511, 2512, 2513, 2514, 2515, 2516, 2517, 2518, 2519, 2520, 2521, 2522, 2523, 2524, 2525, 2526, 2527, 2528, 2529, 2530, 2531, 2532, 2533, 2534, 2535, 2536, 2537, 2538, 2539, 2540, 2541, 2542, 2543, 2544, 2545, 2546, 2547, 2548, 2549, 2550, 2551, 2552, 2553, 2554, 2555, 2556, 2557, 2558, 2559, 2560, 2561, 2562, 2563, 2564, 2565, 2566, 2567, 2568, 2569, 2570, 2571, 2572, 2573, 2574, 2575, 2576, 2577, 2578, 2579, 2580, 2581, 2582, 2583, 2584, 2585, 2586, 2587, 2588, 2589, 2590, 2591, 2592, 2593, 2594, 2595, 2596, 2597, 2598, 2599, 2600, 2601, 2602, 2603, 2604, 2605, 2606, 2607, 2608, 2609, 2610, 2611, 2612, 2613, 2614, 2615, 2616, 2617, 2618, 2619, 2620, 2621, 2622, 2623, 2624, 2625, 2626, 2627, 2628, 2629, 2630, 2631, 2632, 2633, 2634, 2635, 2636, 2637, 2638, 2639, 2640, 2641, 2642, 2643, 2644, 2645, 2646, 2647, 2648, 2649, 2650, 2651, 2652, 2653, 2654, 2655, 2656, 2657, 2658, 2659, 2660, 2661, 2662, 2663, 2664, 2665, 2666, 2667, 2668, 2669, 2670, 2671, 2672, 2673, 2674, 2675, 2676, 2677, 26

1. The first step is to identify the problem or question that needs to be answered. This involves understanding the context and the specific requirements of the task.

$$(65) \quad \frac{\partial X_j}{\partial I_{ij}} = \beta_{ij} \frac{X_j}{I_{ij}}$$

and write the first-order conditions as

$$\begin{aligned} (66) \text{ through } (69) \quad & \beta_{11}(\beta_{11} + \beta_{21})X_1/I_{11} = \beta_{12}(\beta_{12} + \beta_{22})P_2X_2/(P_1I_{12}) \\ & = \beta_{21}(\beta_{11} + \beta_{21})P_1X_1/(P_2I_{21}) = \beta_{22}(\beta_{12} + \beta_{22})X_2/I_{22} \\ & = \frac{\lambda[r - (g_F + g_w)]}{1 + \lambda(1 - c)[r - (g_F + g_w)]} \end{aligned}$$

That the second-order conditions are satisfied is demonstrated

Figure 1



(10)

The following table shows the results of the simulation.

Table 1. Simulation results for the reaction network.

$$\begin{aligned}
 & \frac{dA}{dt} = k_1 B + k_2 C - k_3 A \\
 & \frac{dB}{dt} = k_4 A - k_5 B \\
 & \frac{dC}{dt} = k_6 B - k_7 C
 \end{aligned}$$

$$\frac{dA}{dt} = k_1 B + k_2 C - k_3 A$$



The graph shows the time evolution of the concentrations of species A, B, and C. The x-axis represents time (t) and the y-axis represents concentration. Species A starts at a high concentration and decreases towards zero. Species B starts at zero, increases to a peak, and then decreases. Species C starts at zero and increases monotonically.

The following table shows the results of the simulation.

in Appendix I.

3. Solving for Industry Revenues $P_j X_j$

Use the first-order conditions (66) through (68) to express I_{12} in terms of I_{11} and I_{21} in terms of I_{22} . Insert the results into (40) and (41). Insert (59) into (38) and (39). Insert the results into (40) and (41). Divide (40) by $\beta_{11}(\beta_{11} + \beta_{21})X_1$ and (41) by $\beta_{22}(\beta_{12} + \beta_{22})X_2$, deduct (41) from (40), and again use the first-order conditions (66) through (68). Now define

$$(70) \quad \rho \equiv (P_1 X_1 / (P_2 X_2))$$

rearrange, and write the quadratic

$$(71) \quad \rho^2 + m\rho + n = 0$$

where m and n are the following agglomerations of taste and

technology parameters

$$m \equiv \frac{(\beta_{12} + \beta_{22})[\beta_{12}\pi_2 + \beta_{22}(1 - \pi_1)] - (\beta_{11} + \beta_{21})[\beta_{11}(1 - \pi_2) + \beta_{21}\pi_1]}{(\beta_{11} + \beta_{21})[\beta_{11}\pi_2 + \beta_{21}(1 - \pi_1)]}$$

$$n \equiv - \frac{(\beta_{12} + \beta_{22})[\beta_{12}(1 - \pi_2) + \beta_{22}\pi_1]}{(\beta_{11} + \beta_{21})[\beta_{11}\pi_2 + \beta_{21}(1 - \pi_1)]}$$

The quadratic has the two roots

$$\rho = -m/2 \pm \sqrt{(m/2)^2 - n}$$

We have assumed that $0 < A_i < 1$, $0 < \beta_i < 1$, and $0 < c < 1$, hence $n < 0$. Now regardless of the sign of m , $0 \leq (m/2)^2$, hence

$$0 \leq (m/2)^2 < (m/2)^2 - n$$

Two things follow. First, from $0 < (m/2)^2 - n$ it follows that both roots are real. Second, from $(m/2)^2 < (m/2)^2 - n$ it follows that regardless of the sign of m , the first root is positive and the second negative. We reject the latter and are left with

$$(72) \quad p = -m/2 + \sqrt{(m/2)^2 - n}$$

Use (24), (25), (35), and (36) to find

$$\alpha_1 P_1 X_1 + \alpha_2 P_2 X_2 = wF$$

Take this together with (70) and find

$$n = 10$$

$$n = 10$$

$$n = 10 \quad (10 \times 10) = 100$$

$$n = 10 \quad (10 \times 10) = 100$$

1. The first part of the problem is to find the value of n for which the sum of the first n terms of the arithmetic progression is equal to 100. The sum of the first n terms of an arithmetic progression is given by the formula $S_n = \frac{n}{2}(2a + (n-1)d)$, where a is the first term and d is the common difference. In this case, $a = 1$ and $d = 1$. So, we have $S_n = \frac{n}{2}(2 + (n-1)) = \frac{n}{2}(n+1)$. We need to find n such that $S_n = 100$. This gives us the equation $\frac{n}{2}(n+1) = 100$, which simplifies to $n(n+1) = 200$. This is a quadratic equation, $n^2 + n - 200 = 0$. Solving this equation, we get $n = 14$ or $n = -15$. Since n must be a positive integer, we have $n = 14$.

$$n = 10 \quad (10 \times 10) = 100$$

$$n = 10 \quad (10 \times 10) = 100$$

2. The second part of the problem is to find the value of n for which the sum of the first n terms of the arithmetic progression is equal to 100. The sum of the first n terms of an arithmetic progression is given by the formula $S_n = \frac{n}{2}(2a + (n-1)d)$, where a is the first term and d is the common difference. In this case, $a = 1$ and $d = 1$. So, we have $S_n = \frac{n}{2}(2 + (n-1)) = \frac{n}{2}(n+1)$. We need to find n such that $S_n = 100$. This gives us the equation $\frac{n}{2}(n+1) = 100$, which simplifies to $n(n+1) = 200$. This is a quadratic equation, $n^2 + n - 200 = 0$. Solving this equation, we get $n = 14$ or $n = -15$. Since n must be a positive integer, we have $n = 14$.

$$n = 10 \quad (10 \times 10) = 100$$

3. The third part of the problem is to find the value of n for which the sum of the first n terms of the arithmetic progression is equal to 100. The sum of the first n terms of an arithmetic progression is given by the formula $S_n = \frac{n}{2}(2a + (n-1)d)$, where a is the first term and d is the common difference. In this case, $a = 1$ and $d = 1$. So, we have $S_n = \frac{n}{2}(2 + (n-1)) = \frac{n}{2}(n+1)$. We need to find n such that $S_n = 100$. This gives us the equation $\frac{n}{2}(n+1) = 100$, which simplifies to $n(n+1) = 200$. This is a quadratic equation, $n^2 + n - 200 = 0$. Solving this equation, we get $n = 14$ or $n = -15$. Since n must be a positive integer, we have $n = 14$.

$$(73), (74) \quad P_i X_i = \mu_i w F$$

where

$$\mu_1 \equiv \rho / (\alpha_1 \rho + \alpha_2)$$

$$\mu_2 \equiv 1 / (\alpha_1 \rho + \alpha_2)$$

4. Solving for Employments L_i and Income Y

Use (24), (25), (73), (74) to solve for employments

$$(75), (76) \quad L_i = \alpha_i \mu_i F$$

Insert (73) and (74) into (59) and solve for national money income

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1. 1941

1941 = 1941

1941 = 1941

1941 = 1941

1941 = 1941

1941

1941 = 1941

1941 = 1941

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1941 = 1941

1941 = 1941

1941 = 1941

1941 = 1941

1941 = 1941

1941 = 1941

1941 = 1941

1941 = 1941

1941 = 1941

$$(77) \quad Y = (\mu_1 + \mu_2)wF$$

5. Solving for Physical Outputs X_j

Let us begin by finding four investment-output ratios. Again use the first-order conditions (66) through (68) to express I_{12} in terms of I_{11} . Insert the result into (40), insert (59) into (38), and insert the result into (40). Divide (40) by X_1 . Use a similar procedure upon (41) and find the four ratios

$$(78) \quad I_{11}/X_1 = v_{11} \equiv \frac{1 - \pi_1 - \pi_1/\rho}{1 + \beta_{12}(\beta_{12} + \beta_{22})/[\rho\beta_{11}(\beta_{11} + \beta_{21})]}$$

$$(79) \quad I_{12}/X_1 = v_{12} \equiv \frac{1 - \pi_1 - \pi_1/\rho}{1 + \rho\beta_{11}(\beta_{11} + \beta_{21})/[\beta_{12}(\beta_{12} + \beta_{22})]}$$

$$T^2 = \frac{4\pi^2}{g} \left(\frac{L}{2\pi} \right)^2$$

(10)

$$T^2 = \frac{4\pi^2}{g} \left(\frac{L}{2\pi} \right)^2$$

Let us consider a simple pendulum of length L and mass m . The period of oscillation is given by $T = 2\pi \sqrt{\frac{L}{g}}$. The frequency of oscillation is given by $f = \frac{1}{T} = \frac{1}{2\pi} \sqrt{\frac{g}{L}}$. The angular frequency is given by $\omega = 2\pi f = \sqrt{\frac{g}{L}}$. The displacement of the pendulum from its equilibrium position is given by $x = L \sin \theta$, where θ is the angular displacement. The restoring force is given by $F = -mg \sin \theta$. For small angles, $\sin \theta \approx \theta$, so the restoring force is approximately $F = -mg \theta$. The equation of motion is $m \ddot{x} = -mg \theta$. Since $x = L \theta$, we have $\ddot{x} = -\frac{g}{L} x$. This is a simple harmonic oscillator with angular frequency $\omega = \sqrt{\frac{g}{L}}$. The period is $T = 2\pi \sqrt{\frac{L}{g}}$.

$$T^2 = \frac{4\pi^2}{g} \left(\frac{L}{2\pi} \right)^2$$

The period of oscillation is given by $T = 2\pi \sqrt{\frac{L}{g}}$. The frequency of oscillation is given by $f = \frac{1}{T} = \frac{1}{2\pi} \sqrt{\frac{g}{L}}$. The angular frequency is given by $\omega = 2\pi f = \sqrt{\frac{g}{L}}$.

$$T^2 = \frac{4\pi^2}{g} \left(\frac{L}{2\pi} \right)^2$$

The period of oscillation is given by $T = 2\pi \sqrt{\frac{L}{g}}$. The frequency of oscillation is given by $f = \frac{1}{T} = \frac{1}{2\pi} \sqrt{\frac{g}{L}}$. The angular frequency is given by $\omega = 2\pi f = \sqrt{\frac{g}{L}}$.

$$(80) \quad I_{21}/X_2 = v_{21} \equiv \frac{1 - \pi_2 - \pi_2 \rho}{1 + \beta_{22}(\beta_{12} + \beta_{22})/[\rho \beta_{21}(\beta_{11} + \beta_{21})]}$$

$$(81) \quad I_{22}/X_2 = v_{22} \equiv \frac{1 - \pi_2 - \pi_2 \rho}{1 + \rho \beta_{21}(\beta_{11} + \beta_{21})/[\beta_{22}(\beta_{12} + \beta_{22})]}$$

Apply (64) to (78) through (81) and find

$$(82) \quad S_{ij} = X_i v_{ij} / g_{Sij}$$

Insert (82) and our solutions (75) and (76) into the production functions (22) and (23), arrive at two equations in the two unknowns X_j , solve them, and find

— 27 —

1. $\frac{1}{2} \left(\frac{1}{2} + \frac{1}{2} \right) = \frac{1}{2}$

2. $\frac{1}{2} \left(\frac{1}{2} + \frac{1}{2} \right) = \frac{1}{2}$

3. $\frac{1}{2} \left(\frac{1}{2} + \frac{1}{2} \right) = \frac{1}{2}$

4. $\frac{1}{2} \left(\frac{1}{2} + \frac{1}{2} \right) = \frac{1}{2}$

5. $\frac{1}{2} \left(\frac{1}{2} + \frac{1}{2} \right) = \frac{1}{2}$

6. $\frac{1}{2} \left(\frac{1}{2} + \frac{1}{2} \right) = \frac{1}{2}$

$$(83) \quad x_1 = (\xi_1^{1 - \beta_{22} \xi_2^{\beta_{21}}})^{\frac{1}{(1 - \beta_{11})(1 - \beta_{22}) - \beta_{12}\beta_{21}F}}$$

$$(84) \quad x_2 = (\xi_1^{\beta_{12} \xi_2^{1 - \beta_{11}}})^{\frac{1}{(1 - \beta_{11})(1 - \beta_{22}) - \beta_{12}\beta_{21}F}}$$

where

$$\xi_1 \equiv M_1(\alpha_1 \mu_1)^{\alpha_1 (v_{11}/g_{S11})^{\beta_{11}} (v_{21}/g_{S21})^{\beta_{21}}}$$

$$\xi_2 \equiv M_2(\alpha_2 \mu_2)^{\alpha_2 (v_{12}/g_{S12})^{\beta_{12}} (v_{22}/g_{S22})^{\beta_{22}}}$$

The reader may convince himself that (83) and (84) are indeed growing at the rates (56) and (57) said they should be.

100

4

the 1990s, the number of people in the world who are under 15 years of age is expected to increase from 1.1 billion to 1.5 billion. The number of people aged 65 and over is expected to increase from 250 million to 450 million. The number of people aged 15 and over is expected to increase from 3.5 billion to 4.5 billion. The number of people aged 15 and over is expected to increase from 3.5 billion to 4.5 billion. The number of people aged 15 and over is expected to increase from 3.5 billion to 4.5 billion.

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6. Solving for Prices P_j

Divide our revenue solutions (73) and (74) by our physical output solutions (83) and (84), respectively, and find

$$(85) \quad P_1 = (\xi_1^{1 - \beta_{22}} \xi_2^{\beta_{21}}) \frac{1}{(1 - \beta_{11})(1 - \beta_{22}) - \beta_{12}\beta_{21}w\mu_1}$$

$$(86) \quad P_2 = (\xi_1^{\beta_{12}} \xi_2^{1 - \beta_{11}}) \frac{1}{(1 - \beta_{11})(1 - \beta_{22}) - \beta_{12}\beta_{21}w\mu_2}$$

Similarly the reader may convince himself that (85) and (86) are indeed growing at the rates (50) and (51) said they should be.

Let $\alpha_1, \alpha_2, \dots, \alpha_n$ be the roots of the equation

$x^n + a_{n-1}x^{n-1} + \dots + a_1x + a_0 = 0$ and let $\beta_1, \beta_2, \dots, \beta_n$ be the roots of the equation

$x^n + b_{n-1}x^{n-1} + \dots + b_1x + b_0 = 0$.

$$\frac{1}{\alpha_1} + \frac{1}{\alpha_2} + \dots + \frac{1}{\alpha_n} = -\frac{a_{n-1}}{a_0} \quad \text{and} \quad \frac{1}{\beta_1} + \frac{1}{\beta_2} + \dots + \frac{1}{\beta_n} = -\frac{b_{n-1}}{b_0} \quad (1)$$

$$\frac{1}{\alpha_1^2} + \frac{1}{\alpha_2^2} + \dots + \frac{1}{\alpha_n^2} = \left(\frac{1}{\alpha_1} + \frac{1}{\alpha_2} + \dots + \frac{1}{\alpha_n} \right)^2 - 2 \left(\frac{1}{\alpha_1\alpha_2} + \frac{1}{\alpha_1\alpha_3} + \dots + \frac{1}{\alpha_{n-1}\alpha_n} \right) \quad (2)$$

Let $\alpha_1, \alpha_2, \dots, \alpha_n$ be the roots of the equation $x^n + a_{n-1}x^{n-1} + \dots + a_1x + a_0 = 0$ and let $\beta_1, \beta_2, \dots, \beta_n$ be the roots of the equation $x^n + b_{n-1}x^{n-1} + \dots + b_1x + b_0 = 0$. Then

7. Capital Stocks S_{ij} and their Marginal Productivities κ_{ij}

With (83) and (84) inserted into it, (82) will be a solution for physical capital stocks S_{ij} . With (82) inserted into them, (26) through (29) will be solutions for the physical marginal productivities of capital

$$(87) \quad \kappa_{ij} = \beta_{ij} \xi_{Sij} / v_{ij}$$

8. Consumption C_i and Income Distribution W and Z

With (77), (85), and (86) inserted into them, (38) and (39) will be solutions for consumption. With (35) inserted into it, (36) will be a solution for the wage bill

$$(88) \quad W = wF$$

With (73) and (74) inserted into them, (30) through (32) will generate the profits bill

$$(89) \quad Z = [(\beta_{11} + \beta_{21})\mu_1 + (\beta_{12} + \beta_{22})\mu_2]wF$$

With (89) inserted into it, (61) will be a solution for present worth.

9. Properties of Solutions

We have now solved for the levels of all variables. Our solutions (78) through (81) for the investment-output ratios and (87) for the physical marginal productivities of capital are stationary. All other solutions for levels are nonstationary, because they contain one or more of our three nonstationary

1. The first of the three main points of the report is that the Commission has found that the Government of the United Kingdom has failed to provide adequate information to the Commission regarding the activities of the British Intelligence Services in the United States.

2. The second of the three main points of the report is that the Commission has found that the British Intelligence Services have engaged in activities which are in violation of the provisions of the Espionage Laws of the United States.

3. The third of the three main points of the report is that the Commission has found that the British Intelligence Services have engaged in activities which are in violation of the provisions of the Espionage Laws of the United States.

III. CONCLUSIONS

The Commission concludes that the British Intelligence Services have engaged in activities which are in violation of the provisions of the Espionage Laws of the United States. The Commission also concludes that the Government of the United Kingdom has failed to provide adequate information to the Commission regarding the activities of the British Intelligence Services in the United States.

parameters, i. e. available labor force F , the multiplicative factor M_i of the production functions, and the money wage rate w .

Are our solutions real and positive? Section IV, 3 found both roots ρ to be real and found one to be positive, the other negative. All solutions (73) through (89), then, are real. Rejecting the negative root we find solutions (73) through (77), (88), and (89) to be obviously positive. Less obviously, so are solutions (78) through (87), as demonstrated in our Appendix II.

A P P E N D I X I

SECOND-ORDER CONDITIONS FOR A MAXIMUM OF EQUATION (62)

Write the bordered Hessian

$$(90) \quad H \equiv \begin{vmatrix} \frac{\partial^2 \phi}{\partial I_{11}^2} & \frac{\partial^2 \phi}{\partial I_{11} \partial I_{12}} & \frac{\partial^2 \phi}{\partial I_{11} \partial I_{21}} & \frac{\partial^2 \phi}{\partial I_{11} \partial I_{22}} & -P_1 \\ \frac{\partial^2 \phi}{\partial I_{12} \partial I_{11}} & \frac{\partial^2 \phi}{\partial I_{12}^2} & \frac{\partial^2 \phi}{\partial I_{12} \partial I_{21}} & \frac{\partial^2 \phi}{\partial I_{12} \partial I_{22}} & -P_1 \\ \frac{\partial^2 \phi}{\partial I_{21} \partial I_{11}} & \frac{\partial^2 \phi}{\partial I_{21} \partial I_{12}} & \frac{\partial^2 \phi}{\partial I_{21}^2} & \frac{\partial^2 \phi}{\partial I_{21} \partial I_{22}} & -P_2 \\ \frac{\partial^2 \phi}{\partial I_{22} \partial I_{11}} & \frac{\partial^2 \phi}{\partial I_{22} \partial I_{12}} & \frac{\partial^2 \phi}{\partial I_{22} \partial I_{21}} & \frac{\partial^2 \phi}{\partial I_{22}^2} & -P_2 \\ -P_1 & -P_1 & -P_2 & -P_2 & 0 \end{vmatrix}$$

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1997, 1998, 1999, 2000, 2001, 2002, 2003, 2004, 2005, 2006, 2007, 2008, 2009, 2010, 2011, 2012, 2013, 2014, 2015, 2016, 2017, 2018, 2019, 2020, 2021, 2022, 2023, 2024, 2025, 2026, 2027, 2028, 2029, 2030, 2031, 2032, 2033, 2034, 2035, 2036, 2037, 2038, 2039, 2040, 2041, 2042, 2043, 2044, 2045, 2046, 2047, 2048, 2049, 2050, 2051, 2052, 2053, 2054, 2055, 2056, 2057, 2058, 2059, 2060, 2061, 2062, 2063, 2064, 2065, 2066, 2067, 2068, 2069, 2070, 2071, 2072, 2073, 2074, 2075, 2076, 2077, 2078, 2079, 2080, 2081, 2082, 2083, 2084, 2085, 2086, 2087, 2088, 2089, 2090, 2091, 2092, 2093, 2094, 2095, 2096, 2097, 2098, 2099, 2100, 2101, 2102, 2103, 2104, 2105, 2106, 2107, 2108, 2109, 2110, 2111, 2112, 2113, 2114, 2115, 2116, 2117, 2118, 2119, 2120, 2121, 2122, 2123, 2124, 2125, 2126, 2127, 2128, 2129, 2130, 2131, 2132, 2133, 2134, 2135, 2136, 2137, 2138, 2139, 2140, 2141, 2142, 2143, 2144, 2145, 2146, 2147, 2148, 2149, 2150, 2151, 2152, 2153, 2154, 2155, 2156, 2157, 2158, 2159, 2160, 2161, 2162, 2163, 2164, 2165, 2166, 2167, 2168, 2169, 2170, 2171, 2172, 2173, 2174, 2175, 2176, 2177, 2178, 2179, 2180, 2181, 2182, 2183, 2184, 2185, 2186, 2187, 2188, 2189, 2190, 2191, 2192, 2193, 2194, 2195, 2196, 2197, 2198, 2199, 2200, 2201, 2202, 2203, 2204, 2205, 2206, 2207, 2208, 2209, 2210, 2211, 2212, 2213, 2214, 2215, 2216, 2217, 2218, 2219, 2220, 2221, 2222, 2223, 2224, 2225, 2226, 2227, 2228, 2229, 2230, 2231, 2232, 2233, 2234, 2235, 2236, 2237, 2238, 2239, 2240, 2241, 2242, 2243, 2244, 2245, 2246, 2247, 2248, 2249, 2250, 2251, 2252, 2253, 2254, 2255, 2256, 2257, 2258, 2259, 2260, 2261, 2262, 2263, 2264, 2265, 2266, 2267, 2268, 2269, 2270, 2271, 2272, 2273, 2274, 2275, 2276, 2277, 2278, 2279, 2280, 2281, 2282, 2283, 2284, 2285, 2286, 2287, 2288, 2289, 2290, 2291, 2292, 2293, 2294, 2295, 2296, 2297, 2298, 2299, 2300, 2301, 2302, 2303, 2304, 2305, 2306, 2307, 2308, 2309, 2310, 2311, 2312, 2313, 2314, 2315, 2316, 2317, 2318, 2319, 2320, 2321, 2322, 2323, 2324, 2325, 2326, 2327, 2328, 2329, 2330, 2331, 2332, 2333, 2334, 2335, 2336, 2337, 2338, 2339, 2340, 2341, 2342, 2343, 2344, 2345, 2346, 2347, 2348, 2349, 2350, 2351, 2352, 2353, 2354, 2355, 2356, 2357, 2358, 2359, 2360, 2361, 2362, 2363, 2364, 2365, 2366, 2367, 2368, 2369, 2370, 2371, 2372, 2373, 2374, 2375, 2376, 2377, 2378, 2379, 2380, 2381, 2382, 2383, 2384, 2385, 2386, 2387, 2388, 2389, 2390, 2391, 2392, 2393, 2394, 2395, 2396, 2397, 2398, 2399, 2400, 2401, 2402, 2403, 2404, 2405, 2406, 2407, 2408, 2409, 2410, 2411, 2412, 2413, 2414, 2415, 2416, 2417, 2418, 2419, 2420, 2421, 2422, 2423, 2424, 2425, 2426, 2427, 2428, 2429, 2430, 2431, 2432, 2433, 2434, 2435, 2436, 2437, 2438, 2439, 2440, 2441, 2442, 2443, 2444, 2445, 2446, 2447, 2448, 2449, 2450, 2451, 2452, 2453, 2454, 2455, 2456, 2457, 2458, 2459, 2460, 2461, 2462, 2463, 2464, 2465, 2466, 2467, 2468, 2469, 2470, 2471, 2472, 2473, 2474, 2475, 2476, 2477, 2478, 2479, 2480, 2481, 2482, 2483, 2484, 2485, 2486, 2487, 2488, 2489, 2490, 2491, 2492, 2493, 2494, 2495, 2496, 2497, 2498, 2499, 2500, 2501, 2502, 2503, 2504, 2505, 2506, 2507, 2508, 2509, 2510, 2511, 2512, 2513, 2514, 2515, 2516, 2517, 2518, 2519, 2520, 2521, 2522, 2523, 2524, 2525, 2526, 2527, 2528, 2529, 2530, 2531, 2532, 2533, 2534, 2535, 2536, 2537, 2538, 2539, 2540, 2541, 2542, 2543, 2544, 2545, 2546, 2547, 2548, 2549, 2550, 2551, 2552, 2553, 2554, 2555, 2556, 2557, 2558, 2559, 2560, 2561, 2562, 2563, 2564, 2565, 2566, 2567, 2568, 2569, 2570, 2571, 2572, 2573, 2574, 2575, 2576, 2577, 2578, 2579, 2580, 2581, 2582, 2583, 2584, 2585, 2586, 2587, 2588, 2589, 2590, 2591, 2592, 2593, 2594, 2595, 2596, 2597, 2598, 2599, 2600, 2601, 2602, 2603, 2604, 2605, 2606, 2607, 2608, 2609, 2610, 2611, 2612, 2613, 2614, 2615, 2616, 2617, 2618, 2619, 2620, 2621, 2622, 2623, 2624, 2625, 2626, 2627, 2628, 2629, 2630, 2631, 2632, 2633, 2634, 2635, 2636, 2637, 2638, 2639, 2640, 2641, 2642, 2643, 2644, 2645, 2646, 2647, 2648, 2649, 2650, 2651, 2652, 2653, 2654, 2655, 2656, 2657, 2658, 2659, 2660, 2661, 2662, 2663, 2664, 2665, 2666, 2667, 2668, 2669, 2670, 2671, 2672, 2673, 2674, 2675, 2676, 2677, 2678, 26

Figure 1. The effect of the concentration of the *Agrobacterium* suspension on the transformation efficiency of *Agrobacterium* strains.

The first derivatives $\partial\phi/\partial I_{ij}$ have already been taken and were of the form (63). It follows from that form that a good many of the second derivatives contained in our Hessian are zero: After inserting (64) into our production functions (22) and (23) we realize that X_j is a function of neither I_{ii} nor I_{ji} where $i \neq j$, hence

$$(91) \quad \frac{\partial X_j}{\partial I_{ii}} = \frac{\partial X_j}{\partial I_{ji}} = \frac{\partial^2 X_j}{\partial I_{ii} \partial I_{ij}} = \frac{\partial^2 X_j}{\partial I_{ji} \partial I_{ij}} = \frac{\partial^2 X_j}{\partial I_{ii} \partial I_{jj}} = \frac{\partial^2 X_j}{\partial I_{ji} \partial I_{jj}} = 0$$

$$(i \neq j)$$

But X_j is a function of I_{ij} and I_{jj} , hence

$$(92) \quad \frac{\partial^2 X_j}{\partial I_{ij} \partial I_{jj}} = \frac{\partial^2 X_j}{\partial I_{jj} \partial I_{ij}} = \frac{\beta_{ij} \beta_{jj} X_j}{I_{ij} I_{jj}} \quad (i \neq j)$$

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1. 2. 3. 4. 5. 6. 7. 8. 9. 10. 11. 12. 13. 14. 15. 16. 17. 18. 19. 20. 21. 22. 23. 24. 25. 26. 27. 28. 29. 30. 31. 32. 33. 34. 35. 36. 37. 38. 39. 40. 41. 42. 43. 44. 45. 46. 47. 48. 49. 50. 51. 52. 53. 54. 55. 56. 57. 58. 59. 60. 61. 62. 63. 64. 65. 66. 67. 68. 69. 70. 71. 72. 73. 74. 75. 76. 77. 78. 79. 80. 81. 82. 83. 84. 85. 86. 87. 88. 89. 90. 91. 92. 93. 94. 95. 96. 97. 98. 99. 100. 101. 102. 103. 104. 105. 106. 107. 108. 109. 110. 111. 112. 113. 114. 115. 116. 117. 118. 119. 120. 121. 122. 123. 124. 125. 126. 127. 128. 129. 130. 131. 132. 133. 134. 135. 136. 137. 138. 139. 140. 141. 142. 143. 144. 145. 146. 147. 148. 149. 150. 151. 152. 153. 154. 155. 156. 157. 158. 159. 160. 161. 162. 163. 164. 165. 166. 167. 168. 169. 170. 171. 172. 173. 174. 175. 176. 177. 178. 179. 180. 181. 182. 183. 184. 185. 186. 187. 188. 189. 190. 191. 192. 193. 194. 195. 196. 197. 198. 199. 200. 201. 202. 203. 204. 205. 206. 207. 208. 209. 210. 211. 212. 213. 214. 215. 216. 217. 218. 219. 220. 221. 222. 223. 224. 225. 226. 227. 228. 229. 230. 231. 232. 233. 234. 235. 236. 237. 238. 239. 240. 241. 242. 243. 244. 245. 246. 247. 248. 249. 250. 251. 252. 253. 254. 255. 256. 257. 258. 259. 260. 261. 262. 263. 264. 265. 266. 267. 268. 269. 270. 271. 272. 273. 274. 275. 276. 277. 278. 279. 280. 281. 282. 283. 284. 285. 286. 287. 288. 289. 290. 291. 292. 293. 294. 295. 296. 297. 298. 299. 300. 301. 302. 303. 304. 305. 306. 307. 308. 309. 310. 311. 312. 313. 314. 315. 316. 317. 318. 319. 320. 321. 322. 323. 324. 325. 326. 327. 328. 329. 330. 331. 332. 333. 334. 335. 336. 337. 338. 339. 340. 341. 342. 343. 344. 345. 346. 347. 348. 349. 350. 351. 352. 353. 354. 355. 356. 357. 358. 359. 360. 361. 362. 363. 364. 365. 366. 367. 368. 369. 370. 371. 372. 373. 374. 375. 376. 377. 378. 379. 380. 381. 382. 383. 384. 385. 386. 387. 388. 389. 390. 391. 392. 393. 394. 395. 396. 397. 398. 399. 400. 401. 402. 403. 404. 405. 406. 407. 408. 409. 410. 411. 412. 413. 414. 415. 416. 417. 418. 419. 420. 421. 422. 423. 424. 425. 426. 427. 428. 429. 430. 431. 432. 433. 434. 435. 436. 437. 438. 439. 440. 441. 442. 443. 444. 445. 446. 447. 448. 449. 450. 451. 452. 453. 454. 455. 456. 457. 458. 459. 460. 461. 462. 463. 464. 465. 466. 467. 468. 469. 470. 471. 472. 473. 474. 475. 476. 477. 478. 479. 480. 481. 482. 483. 484. 485. 486. 487. 488. 489. 490. 491. 492. 493. 494. 495. 496. 497. 498. 499. 500. 501. 502. 503. 504. 505. 506. 507. 508. 509. 510. 511. 512. 513. 514. 515. 516. 517. 518. 519. 520. 521. 522. 523. 524. 525. 526. 527. 528. 529. 530. 531. 532. 533. 534. 535. 536. 537. 538. 539. 540. 541. 542. 543. 544. 545. 546. 547. 548. 549. 550. 551. 552. 553. 554. 555. 556. 557. 558. 559. 560. 561. 562. 563. 564. 565. 566. 567. 568. 569. 570. 571. 572. 573. 574. 575. 576. 577. 578. 579. 580. 581. 582. 583. 584. 585. 586. 587. 588. 589. 590. 591. 592. 593. 594. 595. 596. 597. 598. 599. 600. 601. 602. 603. 604. 605. 606. 607. 608. 609. 610. 611. 612. 613. 614. 615. 616. 617. 618. 619. 620. 621. 622. 623. 624. 625. 626. 627. 628. 629. 630. 631. 632. 633. 634. 635. 636. 637. 638. 639. 640. 641. 642. 643. 644. 645. 646. 647. 648. 649. 650. 651. 652. 653. 654. 655. 656. 657. 658. 659. 660. 661. 662. 663. 664. 665. 666. 667. 668. 669. 670. 671. 672. 673. 674. 675. 676. 677. 678. 679. 680. 681. 682. 683. 684. 685. 686. 687. 688. 689. 690. 691. 692. 693. 694. 695. 696. 697. 698. 699. 700. 701. 702. 703. 704. 705. 706. 707. 708. 709. 710. 711. 712. 713. 714. 715. 716. 717. 718. 719. 720. 721. 722. 723. 724. 725. 726. 727. 728. 729. 730. 731. 732. 733. 734. 735. 736. 737. 738. 739. 740. 741. 742. 743. 744. 745. 746. 747. 748. 749. 750. 751. 752. 753. 754. 755. 756. 757. 758. 759. 760. 761. 762. 763. 764. 765. 766. 767. 768. 769. 770. 771. 772. 773. 774. 775. 776. 777. 778. 779. 780. 781. 782. 783. 784. 785. 786. 787. 788. 789. 790. 791. 792. 793. 794. 795. 796. 797. 798. 799. 800. 801. 802. 803. 804. 805. 806. 807. 808. 809. 810. 811. 812. 813. 814. 815. 816. 817. 818. 819. 820. 821. 822. 823. 824. 825. 826. 827. 828. 829. 830. 831. 832. 833. 834. 835. 836. 837. 838. 839. 840. 84

$$(93) \quad \frac{\partial^2 X_j}{\partial I_{ij}^2} = \beta_{ij}(\beta_{ij} - 1) \frac{X_j}{I_{ij}^2} \quad (i = j \text{ or } i \neq j)$$

Apply (91), (92), and (93) to the Hessian (90). Then try to produce even more zero elements, making the Hessian easier to evaluate. Factor out $\beta_{11}h_1X_1/I_{11}$ from first row; $\beta_{12}h_2X_2/I_{12}$ from second row; $\beta_{21}h_1X_1/I_{21}$ from third row; and $\beta_{22}h_2X_2/I_{22}$ from fourth row, where h was defined as part of (63). Thereby the first four elements of the fifth column become

$$- P_1 I_{11} / (\beta_{11} h_1 X_1),$$

$$- P_1 I_{12} / (\beta_{12} h_2 X_2),$$

$$- P_2 I_{21} / (\beta_{21} h_1 X_1),$$

$$- P_2 I_{22} / (\beta_{22} h_2 X_2)$$

1897

Received of the Treasurer of the University of California

the sum of \$100.00 for the year 1897

for the purpose of the purchase of books for the library of the University of California

for the year 1897

for the year 1897

for the year 1897

for the year 1897

But according to the first-order conditions (66) through (69) those four values are all equal to $-1/\lambda$. Now factor out $1/I_{11}$ from first column, $1/I_{12}$ from second column, $1/I_{21}$ from third column, $1/I_{22}$ from fourth column, and $1/\lambda$ from fifth column.

If to each element of a row is added the corresponding element of another row, the determinant remains unchanged. So factor out (-1) from the first row and add to each element of it the corresponding element of the third row. Factor out (-1) from the second row and add to each element of it the corresponding element of the fourth row.

If to each element of a column is added the corresponding element of another column, the determinant remains unchanged. So add to each element of the third column the corresponding element of the first column. Add to each element of the fourth column the corresponding element of the second column.

By now the Hessian has been transformed into the following very tractable form:

$$H = \frac{\beta_{11}\beta_{12}\beta_{21}\beta_{22}h_1^2h_2^2x_1^2x_2^2}{I_{11}^2I_{12}^2I_{21}^2I_{22}^2\lambda} \times$$

| | | | | |
|--------------|--------------|------------------------|------------------------|----|
| 1 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 |
| β_{11} | 0 | $-\alpha_1$ | 0 | -1 |
| 0 | β_{12} | 0 | $-\alpha_2$ | -1 |
| $-P_1I_{11}$ | $-P_1I_{12}$ | $-P_1I_{11}-P_2I_{21}$ | $-P_1I_{12}-P_2I_{22}$ | 0 |

$$= \frac{\beta_{11}\beta_{12}\beta_{21}\beta_{22}h_1^2h_2^2x_1^2x_2^2}{I_{11}^2I_{12}^2I_{21}^2I_{22}^2\lambda} [\alpha_2(P_1I_{11} + P_2I_{21}) + \alpha_1(P_1I_{12} + P_2I_{22})]$$

— 17 —

Вопросы, касающиеся
взаимоотношений между
государством и обществом

| | | | | |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 |
| 2 | 2 | 2 | 2 | 2 |
| 3 | 3 | 3 | 3 | 3 |
| 4 | 4 | 4 | 4 | 4 |
| 5 | 5 | 5 | 5 | 5 |
| 6 | 6 | 6 | 6 | 6 |
| 7 | 7 | 7 | 7 | 7 |
| 8 | 8 | 8 | 8 | 8 |
| 9 | 9 | 9 | 9 | 9 |

Вопросы, касающиеся
взаимоотношений между
государством и обществом

Вопросы, касающиеся
взаимоотношений между
государством и обществом

Is our Hessian positive, then? Appendix II will demonstrate that all solutions, including those for $P_i I_{ij}$, are positive. To see if λ is positive, write the fifth first-order condition $\partial\phi/\partial\lambda = 0$ and find it to be the constraint (60). Use (66) through (69) to write

$$P_1(I_{11} + I_{12}) + P_2(I_{21} + I_{22}) =$$

$$\frac{1 + \lambda(1 - c)[r - (g_F + g_w)]}{\lambda[r - (g_F + g_w)]} [(\beta_{11} + \beta_{21})^2 P_1 X_1 + (\beta_{12} + \beta_{22})^2 P_2 X_2]$$

Insert (59) and this into the constraint (60), rearrange, and

(1) The function $f(x)$ is defined on the interval $[0, 1]$ by the formula $f(x) = x^2 + 2x + 1$. Find the value of the definite integral $\int_0^1 f(x) dx$.

$$f(x) = x^2 + 2x + 1$$

$$\int_0^1 (x^2 + 2x + 1) dx = \left[\frac{x^3}{3} + x^2 + x \right]_0^1 = \frac{1}{3} + 1 + 1 = \frac{7}{3}$$

The value of the definite integral is $\frac{7}{3}$.

write the latter

$$\lambda = \frac{\psi}{(1 - c)(1 - \psi)[r - (g_F + g_w)]}$$

where

$$\psi \equiv \frac{(\beta_{11} + \beta_{21})^2 P_1 X_1 + (\beta_{12} + \beta_{22})^2 P_2 X_2}{P_1 X_1 + P_2 X_2}$$

It follows from $0 < \psi < 1$ that $\lambda > 0$, hence the Hessian (90) is positive. And now for its principal minors.

1957

$$\frac{1}{2} \left(\frac{1}{2} + \frac{1}{2} \right) = \frac{1}{2} \left(\frac{1}{2} + \frac{1}{2} \right)$$

1958

$$\frac{1}{2} \left(\frac{1}{2} + \frac{1}{2} \right) = \frac{1}{2} \left(\frac{1}{2} + \frac{1}{2} \right)$$

1959

1960

From the Hessian (90) remove successively fourth, third, and second column and row and obtain the bordered 4×4 , 3×3 , and 2×2 principal minors. Their values are respectively

$$- \frac{\beta_{11}\beta_{12}\beta_{21}h_1^2h_2^2X_1^2X_2^2}{I_{11}^2I_{12}^2I_{21}^2\lambda} [(1 - \beta_{12})(P_1I_{11} + P_2I_{21}) + (1 - \beta_{11} - \beta_{21})P_1I_{12}]$$

$$\frac{\beta_{11}\beta_{12}h_1h_2X_1X_2}{I_{11}^2I_{12}^2\lambda} [(1 - \beta_{12})P_1I_{11} + (1 - \beta_{11})P_1I_{12}]$$

$$- \frac{\beta_{11}h_1X_1}{I_{11}^2\lambda} P_1I_{11}$$

The three values are negative, positive, and negative, respectively.

THEORY

Let $f(x)$ be a function defined on the interval $[a, b]$. The function $f(x)$ is said to be continuous at a point x_0 if for every $\epsilon > 0$ there exists a $\delta > 0$ such that for all x in the interval $[a, b]$ satisfying $|x - x_0| < \delta$, we have $|f(x) - f(x_0)| < \epsilon$.

Let $f(x)$ be a function defined on the interval $[a, b]$. The function $f(x)$ is said to be continuous on the interval $[a, b]$ if it is continuous at every point x_0 in the interval $[a, b]$.

Let $f(x)$ be a function defined on the interval $[a, b]$. The function $f(x)$ is said to be uniformly continuous on the interval $[a, b]$ if for every $\epsilon > 0$ there exists a $\delta > 0$ such that for all x, y in the interval $[a, b]$ satisfying $|x - y| < \delta$, we have $|f(x) - f(y)| < \epsilon$.

Let $f(x)$ be a function defined on the interval $[a, b]$. The function $f(x)$ is said to be Lipschitz continuous on the interval $[a, b]$ if there exists a constant $L > 0$ such that for all x, y in the interval $[a, b]$ we have $|f(x) - f(y)| \leq L|x - y|$.

Let $f(x)$ be a function defined on the interval $[a, b]$. The function $f(x)$ is said to be differentiable at a point x_0 if the limit $\lim_{h \rightarrow 0} \frac{f(x_0 + h) - f(x_0)}{h}$ exists.

Let $f(x)$ be a function defined on the interval $[a, b]$. The function $f(x)$ is said to be differentiable on the interval $[a, b]$ if it is differentiable at every point x_0 in the interval $[a, b]$.

Let $f(x)$ be a function defined on the interval $[a, b]$. The function $f(x)$ is said to be twice differentiable on the interval $[a, b]$ if it is differentiable on the interval $[a, b]$ and its derivative $f'(x)$ is also differentiable on the interval $[a, b]$.

Let $f(x)$ be a function defined on the interval $[a, b]$. The function $f(x)$ is said to be n -times differentiable on the interval $[a, b]$ if it is $(n-1)$ -times differentiable on the interval $[a, b]$ and its $(n-1)$ -th derivative $f^{(n-1)}(x)$ is also differentiable on the interval $[a, b]$.

Let $f(x)$ be a function defined on the interval $[a, b]$. The function $f(x)$ is said to be analytic on the interval $[a, b]$ if it can be represented by a power series $\sum_{n=0}^{\infty} \frac{f^{(n)}(x_0)}{n!} (x - x_0)^n$ that converges to $f(x)$ for all x in the interval $[a, b]$.

A P P E N D I X I I

SIGN OF SOLUTIONS (78) THROUGH (87)

Solutions (78) through (87) contain one of the factors v_{ij} . Could those factors be nonpositive? To show that they cannot we prove that our positive root ρ has the following bounds:

$$(94) \quad \pi_1/(1 - \pi_1) < \rho < (1 - \pi_2)/\pi_2$$

Take the first inequality of (94), insert (72), move the term $-(m/2)$ to the other side, and write the inequality

$$\sqrt{(m/2)^2 - n} > m/2 + \pi_1/(1 - \pi_1)$$

Square the inequality, multiply it by $(1 - \pi_1)^2$, and write it

$$- \pi_1^2 - m\pi_1(1 - \pi_1) - n(1 - \pi_1)^2 > 0$$

$$A = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$B = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Let \mathcal{H} be a Hilbert space and let $\mathcal{H}^{\otimes n}$ be the n -fold tensor product of \mathcal{H} with itself. Let $\mathcal{H}^{\otimes 0} = \mathbb{C}$ and let $\mathcal{H}^{\otimes n} = \mathcal{H} \otimes \mathcal{H}^{\otimes (n-1)}$ for $n \geq 1$. Let $\mathcal{H}^{\otimes n}$ be the space of all n -fold tensor products of elements of \mathcal{H} . Let $\mathcal{H}^{\otimes n}$ be the space of all n -fold tensor products of elements of \mathcal{H} .

$$f(x) = \begin{cases} 1 & \text{if } x \in \mathcal{H}^{\otimes n} \\ 0 & \text{if } x \notin \mathcal{H}^{\otimes n} \end{cases} \quad (1.1)$$

Let \mathcal{H} be a Hilbert space and let $\mathcal{H}^{\otimes n}$ be the n -fold tensor product of \mathcal{H} with itself. Let $\mathcal{H}^{\otimes 0} = \mathbb{C}$ and let $\mathcal{H}^{\otimes n} = \mathcal{H} \otimes \mathcal{H}^{\otimes (n-1)}$ for $n \geq 1$. Let $\mathcal{H}^{\otimes n}$ be the space of all n -fold tensor products of elements of \mathcal{H} .

$$f(x) = \begin{cases} 1 & \text{if } x \in \mathcal{H}^{\otimes n} \\ 0 & \text{if } x \notin \mathcal{H}^{\otimes n} \end{cases} \quad (1.2)$$

Let \mathcal{H} be a Hilbert space and let $\mathcal{H}^{\otimes n}$ be the n -fold tensor product of \mathcal{H} with itself. Let $\mathcal{H}^{\otimes 0} = \mathbb{C}$ and let $\mathcal{H}^{\otimes n} = \mathcal{H} \otimes \mathcal{H}^{\otimes (n-1)}$ for $n \geq 1$. Let $\mathcal{H}^{\otimes n}$ be the space of all n -fold tensor products of elements of \mathcal{H} .

$$f(x) = \begin{cases} 1 & \text{if } x \in \mathcal{H}^{\otimes n} \\ 0 & \text{if } x \notin \mathcal{H}^{\otimes n} \end{cases} \quad (1.3)$$

Now insert the definitions of m and n attached to (71), recall that $\pi_1 + \pi_2 = c$, rearrange, and find

$$(1 - c)[(\beta_{11} + \beta_{21})\beta_{11}\pi_1 + (\beta_{12} + \beta_{22})\beta_{12}(1 - \pi_1)] > 0$$

which it is under our assumptions about β_{ij} and π_i .

Then take the second inequality of (94), insert (72), move the term $-(m/2)$ to the other side, and write the inequality

$$\sqrt{(m/2)^2 - n} < m/2 + (1 - \pi_2)/\pi_2$$

Square the inequality, multiply it by π_2^2 , and write it

$$(1 - \pi_2)^2 + m\pi_2(1 - \pi_2) + n\pi_2^2 > 0$$

Insert the definitions of m and n and find

$$(1 - c)[(\beta_{11} + \beta_{21})\beta_{21}(1 - \pi_2) + (\beta_{12} + \beta_{22})\beta_{22}\pi_2] > 0$$

which it is under our assumptions about β_{ij} and π_i .

Now that we have validated (94), take its first inequality, multiply it by $1 - \pi_1$, divide it by ρ , use the definitions (78) and (79) and find

$$v_{11} > 0$$

$$v_{12} > 0$$

Take the second inequality of (94), multiply it by π_2 , use the definitions (80) and (81) and find

$$v_{21} > 0$$

$$v_{22} > 0$$

We conclude that (78) through (87) are indeed positive.

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A P P E N D I X I I I

EMPIRICAL MEASUREMENT OF GROWTH IMBALANCE

Yotopoulos and Lau [6] have examined growth imbalance in 65 countries for the periods 1948-53, 1954-58, and 1950-60. In each country, six sectors were distinguished, i. e. agriculture, mining, manufacturing, construction, electricity-gas-water and "others," including transportation and communication, services, etc.

Modifying the Yotopoulos-Lau notation slightly to make it consistent with our own, let us define

E_i \equiv income elasticity of demand for output of i th sector

G \equiv proportionate rate of growth of gross domestic product in
constant prices

g_{Xi} \equiv proportionate rate of growth of output of i th sector in
constant prices

w_i \equiv share in gross domestic product of value added by i th sector

1914

1914-1915

1914-1915

1914-1915

1914-1915

1914-1915

Yotopoulos-Lau now applied two different concepts of imbalance. First, an index of Samuelson-Solow-von Neumann imbalance defined as

$$(95) \quad V^* \equiv \frac{1}{G} \sqrt{\sum_{i=1}^n \omega_i (g_{Xi} - G)^2}$$

or, in English, the reciprocal of the national real growth rate times the square root of the weighted sum of the squared deviations of sectoral real growth rates from the national real growth rate.

For their entire sample of 65 countries, Yotopoulos-Lau found a rather strong negative correlation between the Samuelson-Solow-von Neumann index of imbalance and the national real growth rate; The coefficient of correlation was -0.322. They also found the most highly developed countries to have have the lowest index of

Die in der vorliegenden Arbeit beschriebenen Untersuchungen sind in der Hauptsache aus dem Jahre 1934 zu datieren. Die Ergebnisse sind in der folgenden Tabelle zusammengefasst.

| Untersuchungsgegenstand | Ergebnis | Bemerkungen |
|-------------------------|----------|-------------|
| 1. Die Wirkung von ... | ... | ... |
| 2. Die Wirkung von ... | ... | ... |

Die Ergebnisse der Untersuchungen sind in der folgenden Tabelle zusammengefasst. Die Tabelle ist in drei Spalten unterteilt. Die erste Spalte enthält die Untersuchungsgegenstände, die zweite Spalte die Ergebnisse und die dritte Spalte die Bemerkungen. Die Untersuchungen sind in der Reihenfolge der Tabelle angeordnet. Die Ergebnisse sind in der zweiten Spalte angegeben. Die Bemerkungen sind in der dritten Spalte angegeben. Die Untersuchungen sind in der Reihenfolge der Tabelle angeordnet. Die Ergebnisse sind in der zweiten Spalte angegeben. Die Bemerkungen sind in der dritten Spalte angegeben.

INDEX OF SAMUELSON-SOLOW-VON NEUMANN IMBALANCE $V^* \equiv \frac{1}{G} \sqrt{\sum_{i=1}^n \omega_i (g_{Xi} - G)^2}$

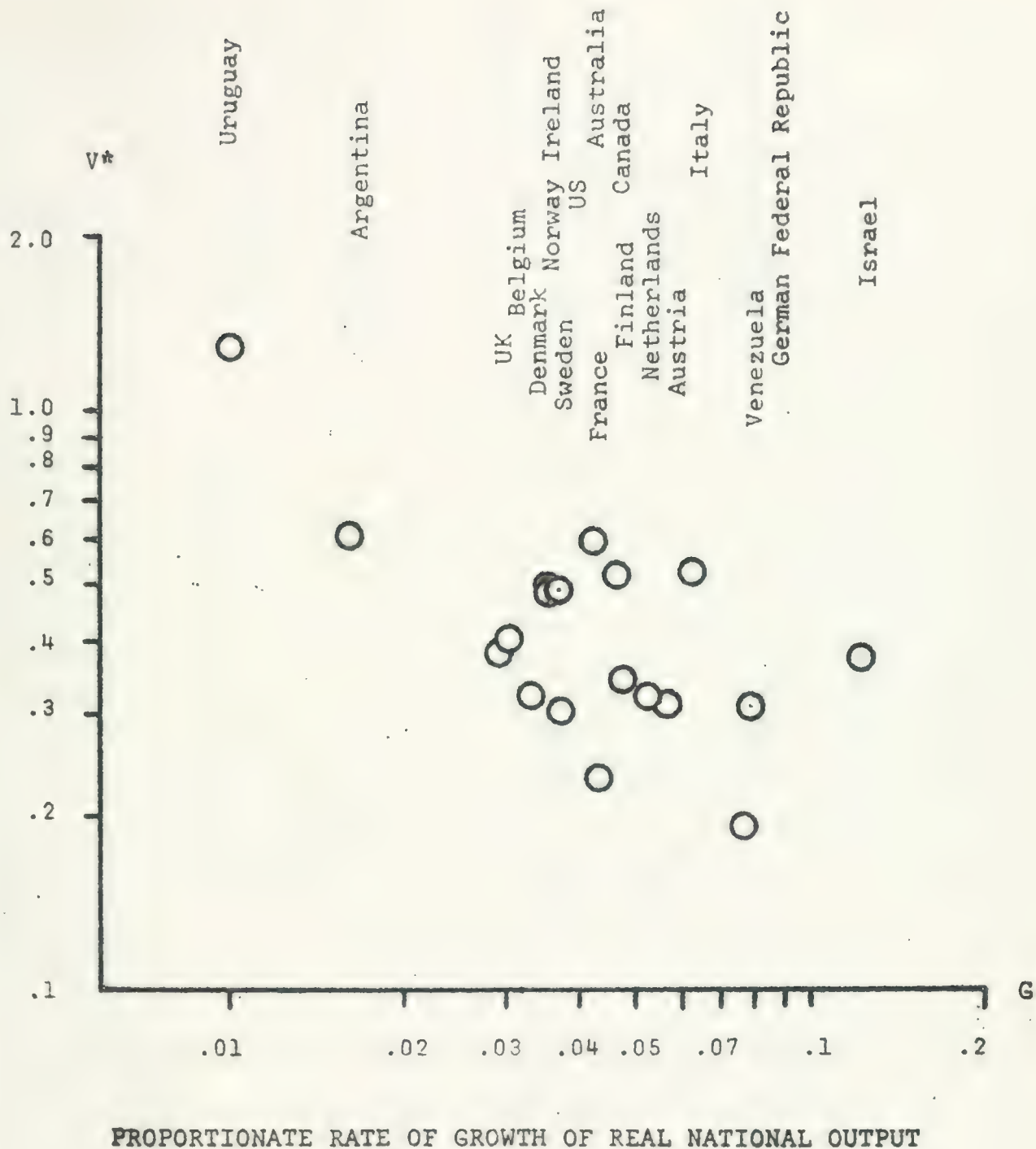


FIGURE 2. SAMUELSON-SOLOW-VON NEUMANN IMBALANCE IN 19 COUNTRIES 1950-60

$$\text{INDEX OF NURKSE IMBALANCE } V' \equiv \frac{1}{G} \sqrt{\sum_{i=1}^n \omega_i (g_{Xi} - E_i G)^2}$$

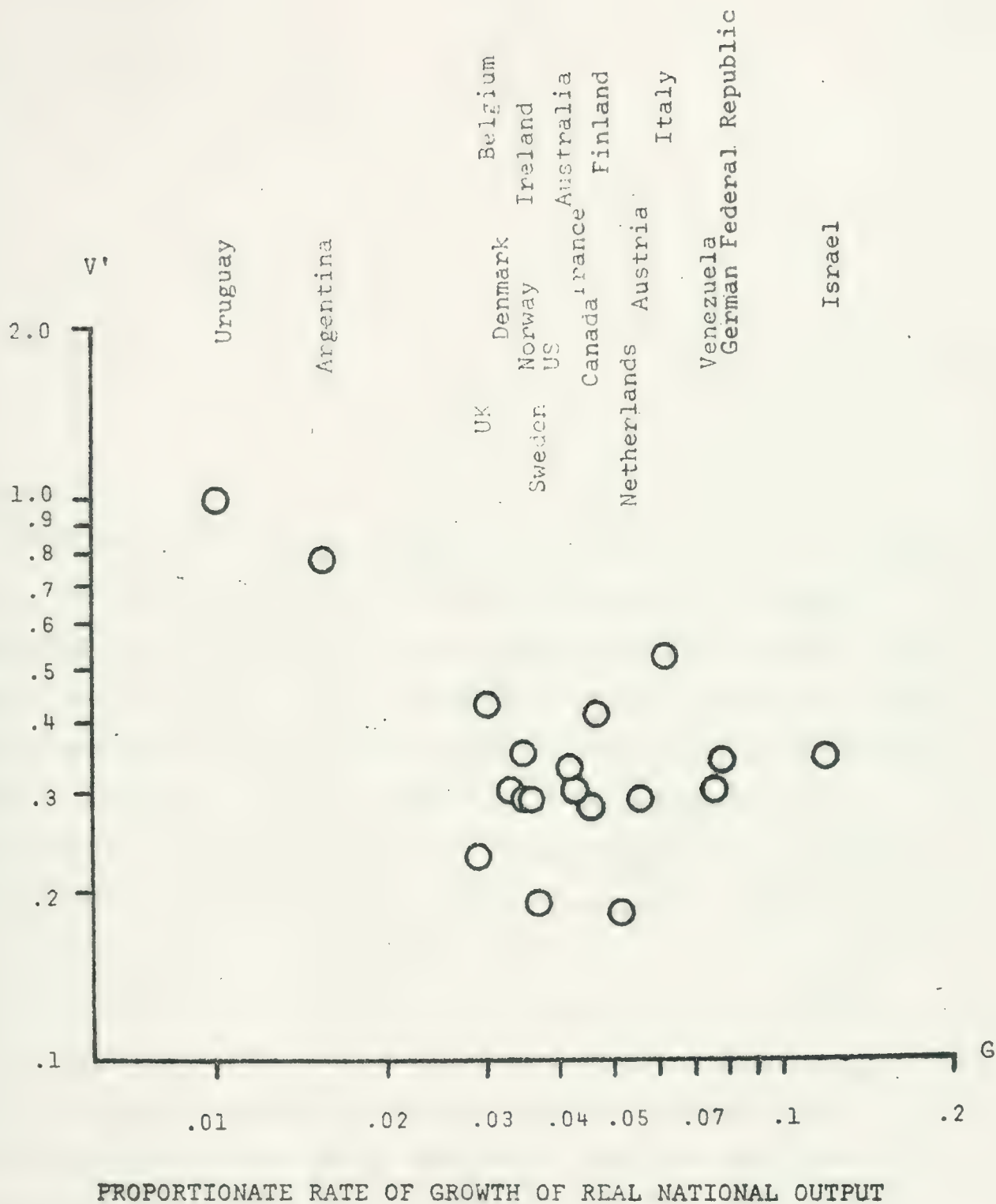


FIGURE 3. NURKSE IMBALANCE IN 19 COUNTRIES 1950-60

imbalance.

From the Yotopoulos-Lau sample of 65 countries our own Figure 2 has selected, for the period 1950-60, a much smaller sample consisting of the 19 capitalist countries which had, in 1958, a per capita income of \$500 or more per annum. Figure 2 shows that even those countries still had a substantial Samuelson-Solow-von Neumann index of imbalance: Their square root of the weighted sum of squared deviations ranged from 0.19 (Venezuela) to 1.26 (Uruguay) of the national real growth rate, with the majority of the countries lying between 0.30 and 0.55 of that rate.

Could imbalance be explained by nonunitary sector income elasticities? Here it occurred to Yotopoulos-Lau to define a second index of imbalance removing from the imbalance concept those deviations which are caused by nonunitary sector income elasticities. That index they called a Nurkse imbalance index and defined it as

...the first of the ...
...the second of the ...
...the third of the ...
...the fourth of the ...
...the fifth of the ...
...the sixth of the ...
...the seventh of the ...
...the eighth of the ...
...the ninth of the ...
...the tenth of the ...
...the eleventh of the ...
...the twelfth of the ...
...the thirteenth of the ...
...the fourteenth of the ...
...the fifteenth of the ...
...the sixteenth of the ...
...the seventeenth of the ...
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...the nineteenth of the ...
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...the twenty-eighth of the ...
...the twenty-ninth of the ...
...the thirtieth of the ...
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...the thirty-eighth of the ...
...the thirty-ninth of the ...
...the fortieth of the ...
...the forty-first of the ...
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...the forty-seventh of the ...
...the forty-eighth of the ...
...the forty-ninth of the ...
...the fiftieth of the ...
...the fifty-first of the ...
...the fifty-second of the ...
...the fifty-third of the ...
...the fifty-fourth of the ...
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...the fifty-sixth of the ...
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...the fifty-eighth of the ...
...the fifty-ninth of the ...
...the sixtieth of the ...
...the sixty-first of the ...
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...the seventy-eighth of the ...
...the seventy-ninth of the ...
...the eightieth of the ...
...the eighty-first of the ...
...the eighty-second of the ...
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...the eighty-eighth of the ...
...the eighty-ninth of the ...
...the ninetieth of the ...
...the ninety-first of the ...
...the ninety-second of the ...
...the ninety-third of the ...
...the ninety-fourth of the ...
...the ninety-fifth of the ...
...the ninety-sixth of the ...
...the ninety-seventh of the ...
...the ninety-eighth of the ...
...the ninety-ninth of the ...
...the hundredth of the ...

$$(96) \quad V' \equiv \frac{1}{G} \sqrt{\sum_{i=1}^n \omega_i (g_{Xi} - E_i G)^2}$$

or, in English, the reciprocal of the national real growth rate times the square root of the weighted sum of the squared deviations of sectoral real growth rates from the product of sector income elasticity and national real growth rate.

Now suppose that imbalance were fully explained by nonunitary sector income elasticities. Then the output of the i th sector would always be growing at the rate $g_{Xi} = E_i G$, consequently according to (96) $V' = 0$. In other words, Nurkse imbalance would be zero.

Applying to the same period and the same countries as Figure 2, our Figure 3 shows that Nurkse imbalance is far from zero. The

Nurkse imbalance in Figure 3 is almost as substantial as the Samuelson-Solow-von Neumann imbalance in Figure 2. The Nurkse range has the same floor but a slightly lower ceiling than the Samuelson-Solow-von Neumann range: The square root of the weighted sum of squared Nurkse deviations ranges from 0.19 (the Netherlands) to 1.0 (Uruguay) of the national real growth rate, with a majority of the countries lying between 0.25 and 0.50 of that rate. We conclude that the Nurkse index has removed precious little imbalance from the Samuelson-Solow-von Neumann index.

How come, so little? Suppose all sector income elasticities were unity, then the Samuelson-Solow-von Neumann index would become equal to the Nurkse index: If $E_i = 1$ it follows from (95) and (96) that $V^* = V'$. And indeed the income elasticities used by Yotopoulos-Lau differed very little from unity:

1. Einleitung

Die vorliegende Arbeit beschäftigt sich mit der Analyse der Auswirkungen der Digitalisierung auf den Arbeitsmarkt. Im Zentrum stehen die Veränderungen in der Nachfrage nach Qualifikationen und die damit verbundenen Herausforderungen für die Bildungssysteme. Ein besonderer Fokus liegt auf der Rolle der Weiterbildung in der Anpassung der Arbeitskräfte an die sich rasch verändernden Anforderungen der Wirtschaft.

Im ersten Teil wird der aktuelle Stand der Forschung zur Digitalisierung des Arbeitsmarktes dargestellt. Es werden die wichtigsten Trends und die daraus resultierenden Konsequenzen für die Beschäftigten diskutiert. Im zweiten Teil wird ein theoretisches Modell entwickelt, das die Zusammenhänge zwischen Digitalisierung, Qualifikationsanforderungen und Bildungsinvestitionen verdeutlicht. Abschließend werden empirische Befunde zur Wirkung von Weiterbildungsmaßnahmen auf die Beschäftigtenpräsentation und die Produktivität analysiert.

Die Ergebnisse zeigen, dass die Digitalisierung zu einer deutlichen Verschiebung der Nachfrage nach hochqualifizierten Arbeitskräften führt. Dies hat erhebliche Auswirkungen auf die Bildungssysteme, die darauf ausgerichtet sein müssen, die entsprechenden Kompetenzen zu vermitteln. Insbesondere die berufliche Weiterbildung spielt eine zentrale Rolle bei der Anpassung der Arbeitskräfte an die neuen Anforderungen. Die Studie liefert wertvolle Erkenntnisse für die Gestaltung von Weiterbildungsprogrammen und die Entwicklung von Personalstrategien in Unternehmen.

| | |
|-----------------------|-------|
| Agriculture | 0.952 |
| Mining | 0.892 |
| Manufacturing | 1.044 |
| Construction | 1.035 |
| Electricity-gas-water | 1.045 |
| Others | 0.999 |

These sector income elasticities were estimated from cross sections of some of the countries examined but applied to all countries.

From the Yotopoulos-Lau measurements we conclude three things. First, that growth imbalance is a rather ubiquitous phenomenon. Second, that in highly developed countries it is not strongly correlated with the national real growth rate. Third, that nonunitary sector income elasticities play a minuscule role in explaining real-world growth imbalance.

F O O T N O T E S

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¹We define, as Hahn and Matthews [3] did, steady-state growth as stationary proportionate rates of growth of physical outputs. We define, as Solow and Samuelson [4] did, balanced growth as identical proportionate rates of growth of physical output for all goods.

²Our Graham-type demand functions (38) and (39) have unitary income elasticities. In our model, then, possible growth imbalance must have causes other than nonunitary income elasticities. From Yotopoulos-Lau [6] one may conclude that nonunitary sector income elasticities play a minuscule role in explaining real-world growth imbalance. This conclusion is derived in Appendix III.

CHAPTER 1

The first chapter of the book is devoted to a general introduction to the subject of the book. It begins with a discussion of the importance of the subject and the scope of the book. It then goes on to discuss the history of the subject and the various methods used to study it. The chapter concludes with a summary of the main points discussed.

The second chapter is devoted to a detailed discussion of the first method used to study the subject. It begins with a description of the method and then goes on to discuss its strengths and weaknesses. The chapter concludes with a summary of the main points discussed.

The third chapter is devoted to a detailed discussion of the second method used to study the subject. It begins with a description of the method and then goes on to discuss its strengths and weaknesses. The chapter concludes with a summary of the main points discussed.

The fourth chapter is devoted to a detailed discussion of the third method used to study the subject. It begins with a description of the method and then goes on to discuss its strengths and weaknesses. The chapter concludes with a summary of the main points discussed.

The fifth chapter is devoted to a detailed discussion of the fourth method used to study the subject. It begins with a description of the method and then goes on to discuss its strengths and weaknesses. The chapter concludes with a summary of the main points discussed.

The sixth chapter is devoted to a detailed discussion of the fifth method used to study the subject. It begins with a description of the method and then goes on to discuss its strengths and weaknesses. The chapter concludes with a summary of the main points discussed.

The seventh chapter is devoted to a detailed discussion of the sixth method used to study the subject. It begins with a description of the method and then goes on to discuss its strengths and weaknesses. The chapter concludes with a summary of the main points discussed.

The eighth chapter is devoted to a detailed discussion of the seventh method used to study the subject. It begins with a description of the method and then goes on to discuss its strengths and weaknesses. The chapter concludes with a summary of the main points discussed.

The ninth chapter is devoted to a detailed discussion of the eighth method used to study the subject. It begins with a description of the method and then goes on to discuss its strengths and weaknesses. The chapter concludes with a summary of the main points discussed.

The tenth chapter is devoted to a detailed discussion of the ninth method used to study the subject. It begins with a description of the method and then goes on to discuss its strengths and weaknesses. The chapter concludes with a summary of the main points discussed.

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GREEK LETTERS USED

α alpha
 β beta
 ζ zeta
 κ kappa
 λ lambda
 μ mu
 ν nu
 ξ xi
 π pi
 ρ rho
 Σ sigma
 τ tau
 ϕ phi
 ψ psi
 ω omega

MATHEMATICAL SYMBOLS USED

{ } brace
[] bracket
| | determinant
= equal to
> greater than
 \equiv identically equal to
 \int integral of
< less than
 \neq not equal to
() parenthesis
 ∂ partial derivative of
 $\sqrt{\quad}$ square root of

Table 1

| Item | Value |
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| 1. Item 1 | 1.00 |
| 2. Item 2 | 2.00 |
| 3. Item 3 | 3.00 |
| 4. Item 4 | 4.00 |
| 5. Item 5 | 5.00 |
| 6. Item 6 | 6.00 |
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| 24. Item 24 | 24.00 |
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| 26. Item 26 | 26.00 |
| 27. Item 27 | 27.00 |
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| 67. Item 67 | 67.00 |
| 68. Item 68 | 68.00 |
| 69. Item 69 | 69.00 |
| 70. Item 70 | 70.00 |
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| 84. Item 84 | 84.00 |
| 85. Item 85 | 85.00 |
| 86. Item 86 | 86.00 |
| 87. Item 87 | 87.00 |
| 88. Item 88 | 88.00 |
| 89. Item 89 | 89.00 |
| 90. Item 90 | 90.00 |
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| 92. Item 92 | 92.00 |
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